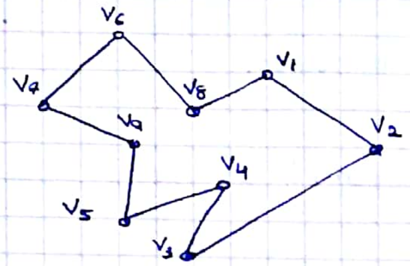
29/Apr/2018. GRAPHS,

(G, V, E) where V = set of ~~vectors~~ vertices
E = set of edges

Eg:



v_1, v_2 are adjacent (connected by one line segment) $\therefore$ $v_1 - v_2$ (edge)
v_1, v_3 are not adjacent

$v_1 - v_2 - v_3$ is a path (walk) $\rightarrow$ path of length 2 (bc. 2 edges)
$v_1 - v_8 - v_6 - v_7 - v_9 - v_5 - v_4 - v_3$ $\Rightarrow$ path of length 7

$d(v_1, v_3)$ = distance from v_1 to v_3 = length of shortest path betw. them.
$\therefore d(v_1, v_3) = 2$

eg: $d(v_1, v_8) = 1$
$d(v_2, v_9) = ?$ Possibilities are: $v_2 - v_4 - v_3 - v_5 - v_9 = 4$
$v_2 - v_1 - v_8 - v_6 - v_7 - v_9 = 5$
$\therefore d(v_2, v_9) = 4.$

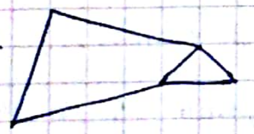
diameter of graph - $diam(G)$ = max. of all distances between every 2 ~~distan.~~ vertices. i.e. calculate distance betw. every two vertices (need not be adjacent). Then take maximum.

eg $d(v_3, v_9) = 3$
$d(v_2, v_9) = 4$
$\vdots$

seems $diam(G) = 4.$

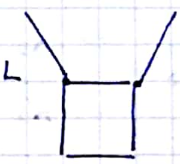
Girth of graph: Length of shortest cycle. In this example, shortest cycle = 1 cycle
$\therefore$ girth $= gr(G) = 9$

Eg: G.



$gr(G) = 3$

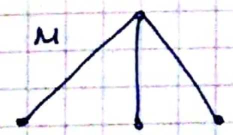
L



$gr(L) = 4$

if the graph has no cycle, then $gr(G) = \infty$

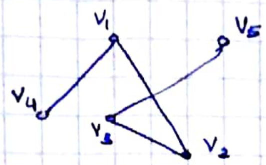
eg: M



$gr(M) = \infty$

Def: We say (G, V, E) is connected if there is a path between every 2 ~~path~~. Vertices.

eg:

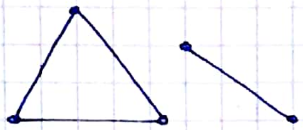


W.

(W, V, E) is connected.

* every edge is a path, but not every path is an edge.

eg:

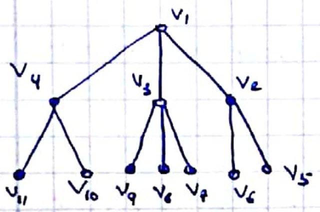


T.

(T, V, E) is disconnected. (at least 2 vertices are not connected by path)

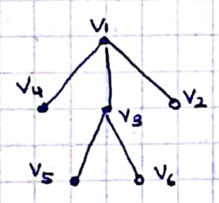
Def: A connected graph is called a tree if it has girth $= \infty$ (no cycles)

Eg:



$|V| = 11$
$|E| = 10$

eg:



$|V| = 6$
$|E| = 5$

Facts (Must know about trees).

- Always connected
- $|E| = |V| - 1$
- Very crucial fact: Between every 2 vertices, there is a unique path.
  $\forall a, b \in G, \exists!$ path.

Def: A connected graph is called complete if every two vertices are connected by an edge.

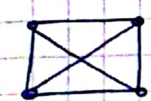
Eg:

$\rightarrow K_3$ (a complete graph w/ 3 vertices).

$\therefore$ 'K_n' means a complete graph w/ n vertices, $n \in \mathbb{N}$.

eg:

$\rightarrow$ connected, but not complete.



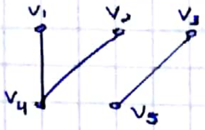
$\rightarrow$ complete.

Def: degree (v_i) = # of edges that meet at v_i, $v_i \in V$

Result: consider K_n. The degree of each vertex is $n-1$

Def: Bi-partite graph.
Consists of 2 sets of vertices, V_1 & V_2, and there is no edge between every 2 vertices within V_i ($i=1$ or $i=2$).

Eg:

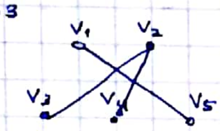


$V_1 = \{v_1, v_2, v_3\}$

$V_2 = \{v_4, v_5\}$

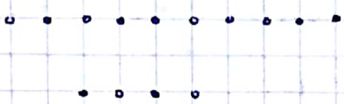
Def: $B_{n,m}$ → bi partite graph, $|V| = n+m$, $|V_1| = n$, $|V_2| = m$

eg: $B_{2,3}$



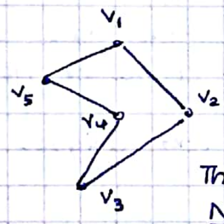
$|V| = 5$

eg: $B_{10,4}$ $|V| = 14$



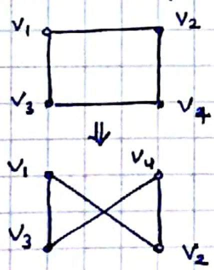
Result: A graph is a bi-partite iff it contains no odd cycles.

eg



This graph is C_5 (cycle with 5 vertices)
Not a bipartite.

eg C_4 is a bipartite

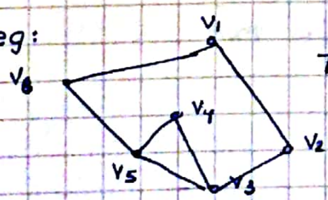


is a bi-partite. Draw C_4 as a bi partite.

$B_{2,2}$ and C_4 are same graph.

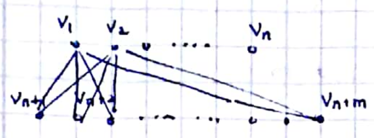
Fact: C_n, where n is even, is always a bi partite.

eg:



This is not a bi partite bc $\exists$ C_3 ($v_4 - v_3 - v_5$).

Def: $K_{n,m}$ → Complete bi-partite graph.



each vertex in V_1 connected to every vertex in V_2 by an edge

By definition, $K_{n,m}$ will have no odd cycles (bc. bipartite).
Degree of $V_1 = m$ degree of $V_2 = n$

eg $K_{2,3}$

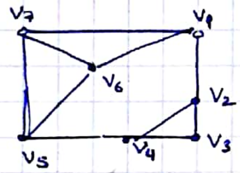
$V_1 = \{v_1, v_2\}$ degree $(v) = 3$ $v \in V_1$

$V_2 = \{v_3, v_4, v_5\}$ degree $(v) = 2$ $v \in V_2$

Result: $\sum_{v \in V} \deg(v) = 2|E|$.

Sum of degrees of all vertices is twice the number of edges

Eg:



$\sum$ degree $(v) = 3+3+3+2+3+3+3$
$= 20$

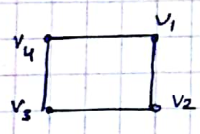
$|E| = 10$.

Observe, $\sum$degree (v) will always be an even number.

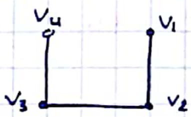
Result: Let D be connected graph. Then D must have a spanning tree, L.
Spanning tree L is sub-graph of D → connected, no cycles, vertices L = vertices D.

i.e. every connected graph must have a spanning tree (not unique)

eg:

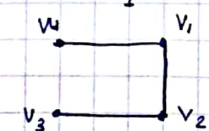


→



D

L_1

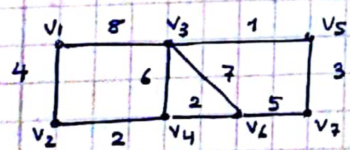


L_2.

and so on.

and between every 2 vertices $\exists!$ path.

Question: Use Dijkstra algorithm to find minimum spanning tree



* weighted graph.

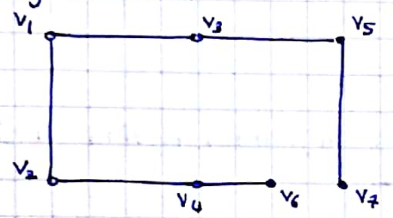
Ans:

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
V_1	$\boxed{0}$	4^{V_1}	8^{V_1}	∞	∞	∞	∞
V_2		$\boxed{4^{V_1}}$	8^{V_1}	6^{V_2}	∞	∞	∞
V_4			8^{V_1}	$\boxed{6^{V_2}}$	∞	8^{V_4}	∞
V_3			$\boxed{8^{V_1}}$		9^{V_3}	8^{V_4}	∞
V_6					9^{V_3}	$\boxed{8^{V_4}}$	13^{V_6}
V_5					$\boxed{9^{V_3}}$		12^{V_5}
V_7							$\boxed{12^{V_5}}$

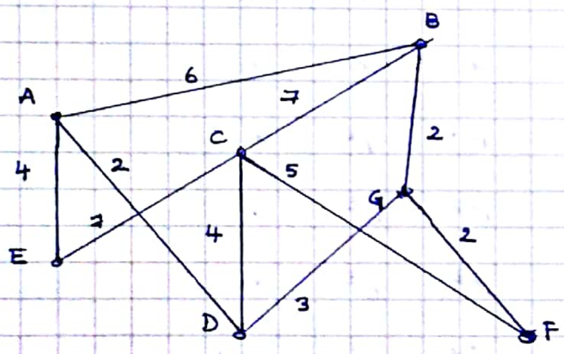
Assume starting from V_1.

done

Drawing the tree.



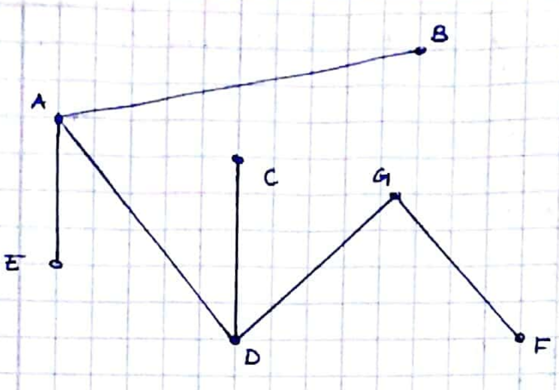
HOMEWORK 13:



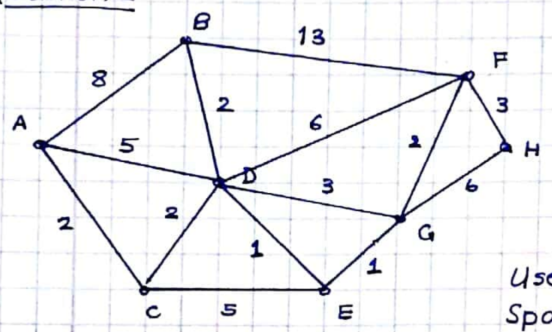
Use Dijkstra's algorithm to find the minimum spanning tree.

	A	B	C	D	E	F	G
A	$\boxed{0}$	6^A	∞	2^A	4^A	∞	∞
D		6^A	6^D	$\boxed{2^A}$	4^A	∞	5^D
E		6^A	6^D		$\boxed{4^A}$	∞	5^D
G		6^A	6^D			7^G	$\boxed{5^D}$
B		$\boxed{6^A}$	6^D			7^G	
C			$\boxed{6^D}$			7^G	
F						$\boxed{7^G}$	

Assume starting from A.

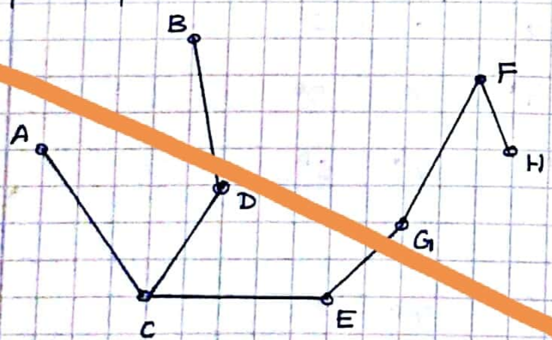


Question 2:



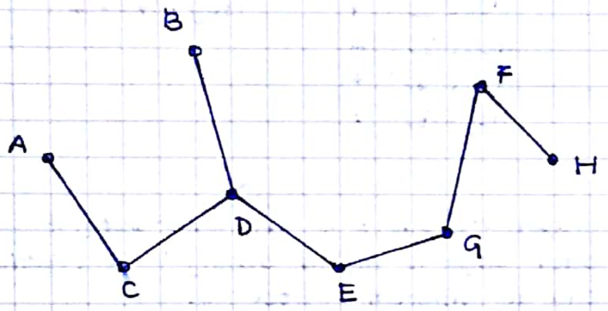
Use Dijkstra's algorithm to find Minimum Spanning tree.

	A	B	C	D	E	F	G	H
A	$\boxed{0}$	8^A	2^A	5^A	∞	∞	∞	∞
C		8^A	$\boxed{2^A}$	4^C	5^C	∞	∞	∞
D		6^D		$\boxed{4^C}$	5^C	10^D	7^D	∞
E		6^D			$\boxed{5^C}$	10^D	6^E	∞
B		$\boxed{6^D}$				10^D	6^E	∞
G						8^G	$\boxed{6^E}$	12^G
F						$\boxed{8^G}$		11^F
H								$\boxed{11^F}$



~~A — B~~

	A	B	C	D	E	F	G	H
A	$\boxed{0}$	8^A	2^A	5^A	∞	∞	∞	∞
C		8^A	$\boxed{2^A}$	4^C	7^C	∞	∞	∞
D		6^D		$\boxed{4^C}$	5^D	10^D	7^D	∞
E		6^D			$\boxed{5^D}$	10^D	6^E	∞
B		$\boxed{6^D}$				10^D	6^E	∞
G						8^G	$\boxed{6^E}$	12^G
F						$\boxed{8^G}$		11^F
H								$\boxed{11^F}$



3-May-2018

Question: degrees of vertices given

Can we construct a graph using the given degrees.

Eg: 4, 3, 2, 1, 1, 1.

Question: Is this sequence graphical? I.e. can you construct a graph from this sequence

First degree : 4.

$S' = 2, 1, 0, 0, 1$ Take one from next 4 vertices

$= 2, 1, 1, 0, 0$ Descending

Second degree : 2

$S'' = 0, 0, 0, 0$

If S^n can be constructed, then the original sequence can be graphically constructed.

Eg: 6, 4, 4, 3, 1, 1, 1, 1. Is this graphical?

$deg(v_1) = 6.$

$S' = 3, 3, 2, 0, 0, 0, 1$

$= 3, 3, 2, 1, 0, 0, 0.$

$deg(v_1') = 3$

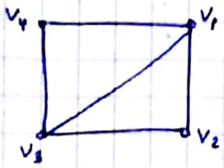
$S'' = 2, 1, 0, 0, 0, 0$

$deg(v_1'') = 2$

$S''' = 0, \underline{-1}, 0, 0, 0.$ graph w/ degree -1 is impossible. Therefore Sequence is not graphical

Definition: Let D be connected graph. We say D is Eulerian (Euler circuit) if we can make a path from a vertex V, s.t. each edge is visited only once (vertices may be visited more than once) and return back to V.

Eg:



not Eulerian

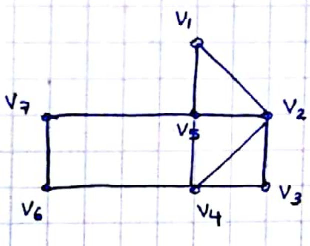
Result: A connected is Eulerian iff degree of each vertex is an even number

eg: $K_{2,3}$ is not Eulerian (vertices in V_1 has degree 3).

eg K_6 is not Eulerian (each vertex has degree $6-1=\underline{5}$).

eg: K_9 is Eulerian (each vertex has degree 8).

Eg:



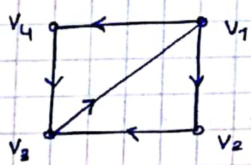
$deg(v_1) = 2$
$deg(v_2) = 4$
$deg(v_3) = 2$
$deg(v_4) = 4$
$deg(v_5) = 4$

$deg(v_6) = 2$
$deg(v_7) = 2$

$\therefore$ Graph is Eulerian.

Def: Let D be a connected graph. We say D is an Euler path. if we can make a path from vertex v s.t. each edge is visited exactly once (vertices may be visited more than once) and end at different vertex y.

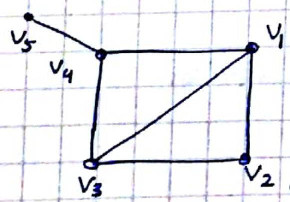
Eg:



is Eulerian path.
$V_1 - V_2 - V_3 - V_1 - V_4 - V_3$

Result: A connected graph is an Euler path iff Exactly 2 vertices are of odd degree.

Eg:

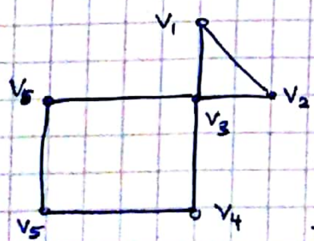


is not an Eulerian path.

$deg(V_1) = 3$ $deg(v_3) = 3$ $deg(v_4) = 3$ $deg(v_5) = 1$.

Definition: Let D be a connected graph. We say D is Hamiltonian (Hamilton cycle) if we can make a path from a vertex V, s.t each vertex in the graph is visited exactly once and then return to V.

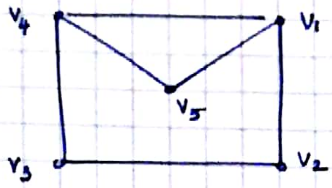
Eg:



Not Hamiltonian.
Is Eulerian.
Is not Euler path.

Traveling salesman / post office ~~pot~~ problem.
Cannot be solved bc graph is not ~~Haltonian~~ Hamiltonian

Eg:



Is hamiltonian.
$V_4 - V_5 - V_1 - V_2 - V_3 - V_4$

Not Eulerian
Is Eulerian path

In hamiltonian graph, there is only one path. There min path = hamiltonian path

6-May-2018 Suppose:
$f(x) = 2x^5 + 7x^3 - 2x^2 + 6$ is a polynomial.
coef are $2, 7, -2, 6 \in \mathbb{Z}$.

Find all rational roots of $f(x)$.
- polynomial of degree 5. Max. 5 roots.

Algorithm: Find all factors of constant : 6.
$\Rightarrow 1, -1, 6, -6, 2, -2, 3, -3.$

Find factors of leading coefficient ($2x^5$):
$\Rightarrow 1, -2, -1, 2$

possible rational roots: $\frac{1}{1}, \frac{1}{-1}, \frac{1}{2}, \frac{1}{-2}$ $\qquad \frac{\text{factor of constant}}{\text{factor of leading constant}}$

$6, -6, 3, -3$
$2, -2, \frac{3}{2}, -\frac{3}{2}$

check $f(1), f(-1), f(\frac{1}{2}), f(\frac{-1}{2}) \dots$

if none of these equal 0, then $f(x)$ has no rational root.

* Definitely coming for final.

Suppose: $x^4 + 2x^3 + 8x + 6$
Assume p is a prime number. Assume p is factor of all coefficients, except leading coefficient. If $p^2 \nmid$ constant, then $f(x)$ has ~~not~~ no rational roots. over $\mathbb{Q}$

Factors of 6: $1, -1, 2, -2, 3, -3, 6, -6$
Factors of 1: $1, -1$.
possible roots: $1, -1, 2, -2, 3, -3, 6, -6$
$p = 2$
$2 \mid 2, \quad 2 \mid 8, \quad 2 \mid 6, \quad 2 \nmid 1$
$2^2 = 4, \quad 4 \nmid 6 \quad \therefore$ $f(x)$ has ~~not~~ no rational roots.

Suppose: $f(x) = 4x^3 + 7x - 14$
claim: $f(x)$ has no roots over ~~$\mathbb{R}$~~ $\mathbb{Q}$
Proof: take $p = 7$
$7 \mid -14, \quad 7 \mid 7, \quad 7 \nmid 4.$
$7^2 = 49. \rightarrow 49 \nmid -14$
$\therefore$ $f(x)$ has no roots over $\mathbb{Q}$.

BRUNNEN

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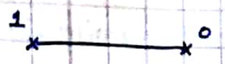
Hyper-cube graph / n-cube graph.

Let $n = 1$.

n-cube graph $\rightarrow$ 1-cube graph. $\Rightarrow Q_1$

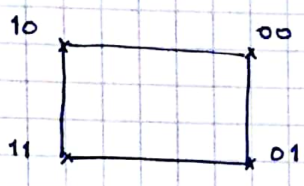
$|V| = 2^n = 2$



let $n = 2$.

2-cube graph. $\Rightarrow Q_2$

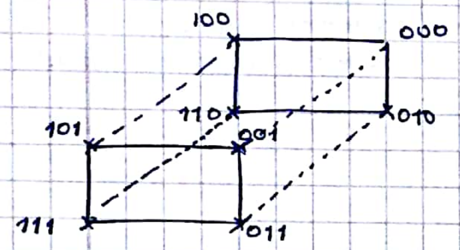
$|V| = 2^2 = 4$ * vertices are connected only when they differ by one digit



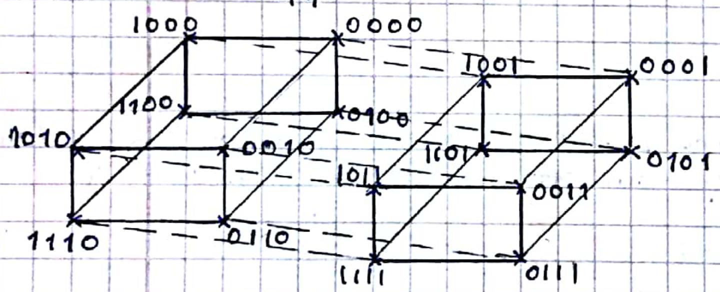
$n = 3$

3-cube $\Rightarrow Q_3$.

$|V| = 8$



4-cube $\Rightarrow Q_4$



Properties: $|V| = 2^n$

degree of each vertex $= n$, if $n \geq 1$.

diameter $= n$

girth $= 4$ if $n \geq 2$, girth $= \infty$ if $n = 1$

graph is bi-partite, when $n \geq 2$.